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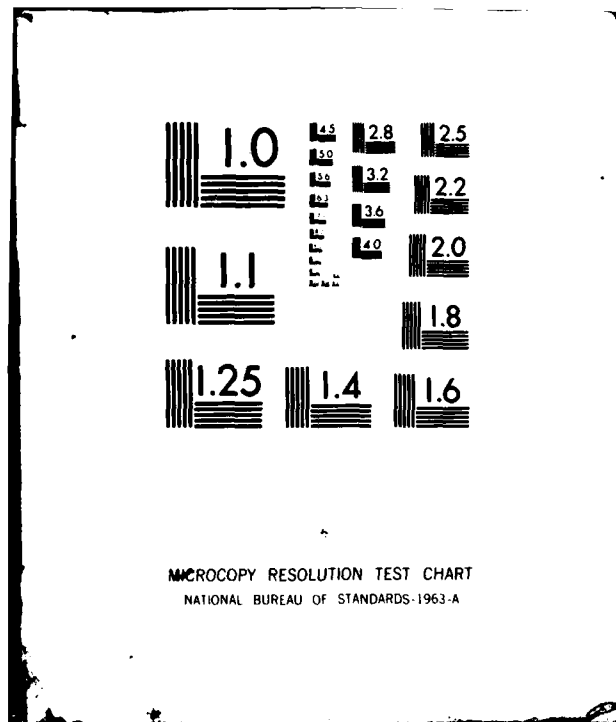
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THE THEORETICAL SIGNAL RESPONSE OF A PASSIVE VERTICAL ARRAY IN --ETC(U)
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9 Memorandum rept.,

6 THE THEORETICAL SIGNAL RESPONSE OF A PASSIVE VERTICAL ARRAY
IN SHALLOW WATER.

by

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11 1 Oct 1979

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This memorandum has been prepared within the SACLANTCEN Underwater Research Division as part of Project 15.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
1 DEFINITIONS	3
2 THE SIGNAL FIELD	6
3 METHODS OF COMBINING HYDROPHONES	7
3.1 Beamforming Method	8
3.2 Mode-Matching Method	9
4 RESULTS: VARIATION OF SIGNAL GAIN WITH ARRAY LENGTH	11
5 EFFECT OF PARAMETERS	13
5.1 Range	14
5.2 Frequency	14
5.3 Bottom Type	17
5.4 Sound-speed Profile	18
5.5 Source Depth	18
5.6 Array Depth	18
5.7 Water Depth	21
5.8 Variation of Hydrophone Spacing	21
CONCLUSIONS	24
ACKNOWLEDGMENT	25
REFERENCES	25

List of Figures

1. Example of Weightings to Match to a Third Order Mode	9
2. Signal Gain vs. Number of Hydrophones	12
3. Effect of Range on Signal Gain	15
4. Effect of Frequency on the Signal Gain of a Half-Wavelength Spaced Array	15
5. Effect of Frequency on the Signal Gain of an Array with Spacing 1.5 m	16
6. Effect of Bottom Conditions	16
7. Effect of Sound Speed Profile	19
8. Effect of Source Depth - Summer Profile	19
9. Effect of Source Depth - Winter Profile	20
10. Effect of Array Depth	20
11. Effect of Water Depth	22
12. Signal Gain for Different Frequency/Water Depth Combinations	22
13. Signal Gain by Beamforming for Different Hydrophone Spacings	23

THE THEORETICAL SIGNAL RESPONSE OF A PASSIVE VERTICAL ARRAY IN SHALLOW WATER

by

R.M. HAMSON

ABSTRACT

↓ The performance of an array of hydrophones is determined by its response to the signal and noise fields in which it operates. This paper investigates the theoretical response of a vertical line array to the signal field in shallow water, taking the noise field as isotropic. A normal-mode propagation model was used to provide the incident field. Two methods of combining hydrophone signals have been studied: a broadside beamforming approach and a mode-matching method. In the first, limitations to the gain of the array are found due to the particular propagation properties of the shallow water environment. The maximum gain is achieved with an array of length 15λ to 20λ . Longer arrays have degraded performance. The second, mode-matching, method involves applying amplitude and phase weightings to the array in order to enhance its response to a selected propagation mode; provided the most powerful mode is chosen, this gives some gain improvements. The effects of the relevant parameters, e.g. range, frequency, bottom conditions, sound-speed profile, etc., on array performance and its variation with array length have been investigated and general conclusions drawn.

LAMBDA ↗

INTRODUCTION

ASW detection improvements are being studied at SACLANTCEN with emphasis on shallow water. In particular, line arrays of hydrophones are being investigated both theoretically and experimentally to determine the limitations imposed on their performances by the signal and noise fields typical of this special environment. The aims of these studies are to identify the physical design parameters (length of array, number of hydrophones), the method of combination of hydrophone signals, and the array position in the water that correspond to best detection performance.

This first interim report is solely concerned with the theoretical study of vertical arrays of equally-spaced hydrophones. The problem can be separated into two areas: response of the array to the signal field and response of the array to the noise field. We consider here in detail

the first half of the problem, taking the noise as isotropic. A separate study will cover the effect of surface (wind-generated) noise.

In an ideal situation where the signal is a plane wave normally incident on the array and the noise is completely incoherent, the performance of an array, by addition of hydrophone signals in phase, increases linearly with the number of hydrophones. Under realistic shallow-water conditions the signal and noise fields are more complicated and array performance will generally be degraded. It will not necessarily increase indefinitely with the addition of more hydrophones.

The main feature of the shallow-water environment that causes these complications is the proximity of the bottom. Wave propagation over any reasonable distance inevitably includes many interactions with the bottom, and is affected by the acoustic properties of the upper layers of the sea bed. Another important factor, which also applies to deep water and has a considerable effect on propagation, is the variation of sound speed with depth.

For the study of the signal response of the array, we consider the acoustic field to be that due to a distant point source of sound, and composed of a discrete number of normal modes of propagation. These modes arise from the solution of the wave equation, taking into account bottom boundary conditions and varying sound speed within the channel. Thus a normal-mode computer model can provide the signal field incident on the array for specified conditions.

Array performance also depends on the method of combining the individual hydrophone signals. We have investigated two approaches: a simple broadside beamforming method (i.e. in phase addition of hydrophone signals), and a mode-matching method. The latter involves applying amplitude and phase weightings to the hydrophones in accordance with the shape of a selected propagation mode.

We measure the array performance in all situations by its gain over a single hydrophone. The hydrophones of the array will in general have different outputs due to the non-homogeneity of the acoustic field. We have used an averaged single-hydrophone output for array-gain calculations. However, the hydrophone with the largest output is also monitored to check for any cases when it has better performance than the array itself. Comparisons of the effects of the relevant parameters (e.g. range, frequency, sound-speed profile, bottom conditions), the physical characteristics of the array and the two processing methods are made on the basis of array gain.

On the experimental side, sea tests have been carried out with a vertical array of ten hydrophones in 120 m of water in an area of known bottom characteristics. These experiments, which involved both signal and noise measurements, are at present being analyzed and will be reported separately. A further trial is underway with a longer array composed of three differently-spaced sub-arrays, each of thirteen elements.

1 DEFINITIONS

Array Gain is defined [1] as the improvement in signal-to-noise ratio obtained by the array over that obtained by a single hydrophone.

$$A.G. = \frac{(\text{signal power/noise power})_{\text{array}}}{(\text{signal power/noise power})_{\text{one hydrophone}}} \quad (\text{Eq. 1})$$

In general this quantity varies with :

- a) The signal and noise fields in which the array is operating, which in turn depend on the sources of signal and noise and the propagation properties of the environment.
- b) The type of processing used — methods of combining hydrophone signals.
- c) The physical characteristics of the array — its length, number of hydrophones, spacings.

Our task is to calculate array gains for various signal and noise fields that may exist in shallow water and, from the results, to select the physical characteristics and processing methods that give the highest gains.

We see from the definition of array gain that we require a reference signal-to-noise ratio for a single hydrophone. In general this will vary from hydrophone to hydrophone and we have chosen to take this reference level as the average of the signal-to-noise power ratios of all hydrophones of the array.

Consider the signal part of Eq. 1 and let v_i = complex signal voltage at the i^{th} hydrophone. Then the signal power output of the array by weighted addition of the N hydrophones is

$$\left(\sum_{i=1}^N w_i v_i \right)^2 ,$$

where w_i = complex weighting applied to the i^{th} hydrophone and satisfies the normalization condition

$$\frac{1}{N} \sum_{i=1}^N w_i^2 = 1 .$$

The 'average hydrophone' signal power is given by

$$\frac{1}{N} \sum_{i=1}^N v_i^2$$

Hence the signal part of Eq. 1 is

$$\frac{(\text{signal power})_{\text{array}}}{(\text{signal power})_{\text{average hydrophone}}} = \frac{\left(\sum_{i=1}^N w_i v_i \right)^2}{\frac{1}{N} \sum_{i=1}^N v_i^2} \quad (\text{Eq. 2})$$

For the noise part of Eq. 1 it is often useful to express the array power due to noise in terms of cross-correlation coefficients between pairs of hydrophones. [1, p. 33].

We have

$$(\text{noise power})_{\text{array}} = \left(\sum_{i=1}^N w_i n_i \right)^2,$$

where n_i is the noise voltage at the i^{th} hydrophone.

This can be written as

$$(\text{noise power})_{\text{array}} = \sum_{i=1}^N \sum_{j=1}^N w_i w_j c_{ij} \quad (\text{Eq. 3})$$

where c_{ij} is the un-normalized cross-correlation coefficient between the i^{th} and j^{th} hydrophones.

As we are at present concerned with the signal part of the problem, we make the assumption in Eq. 3 that the noise is spherically isotropic.

In this case we have equal noise power at each hydrophone ($=n^2$ say), and the normalized correlation coefficient between the i^{th} and j^{th} hydrophones separated by a distance $d(i-j)$ is from [1],

$$P_{ij} = \frac{c_{ij}}{n^2} = \frac{\sin\left(\frac{2\pi d}{\lambda} (i-j)\right)}{\frac{2\pi d}{\lambda} (i-j)}$$

Thus

$$(\text{isotropic noise power})_{\text{array}} = n^2 \left[N + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N w_i w_j \frac{\sin\left(\frac{2\pi d}{\lambda}(i-j)\right)}{\frac{2\pi d}{\lambda}(i-j)} \right]$$

and the noise part of Eq. 1 becomes

$$\frac{(\text{noise power})_{\text{average hydrophone}}}{(\text{noise power})_{\text{array}}} = \frac{1}{N + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N w_i w_j \frac{\sin\left(\frac{2\pi d}{\lambda}(i-j)\right)}{\frac{2\pi d}{\lambda}(i-j)}} \quad (\text{Eq. 4})$$

Hence from Eqs. 1, 2 and 4 we have an expression for the 'signal gain' of the array, G_S , defined as the array gain in isotropic noise conditions

$$G_S = \frac{\left(\sum_{i=1}^N w_i v_i \right)^2}{\frac{1}{N} \sum_{i=1}^N v_i^2} \times \frac{1}{\left[N + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N w_i w_j \frac{\sin\left(\frac{2\pi d}{\lambda}(i-j)\right)}{\frac{2\pi d}{\lambda}(i-j)} \right]} \quad (\text{Eq. 5})$$

G_S reduces to the quantity 'directivity index', D.I., when the w_i 's = 1, and the signal field is a single plane wave, normally incident on the array, i.e. when there is complete coherence between hydrophone signals. In this case we have

$$\text{D.I.} = \frac{N^2}{N + 2 \sum_{\rho=1}^{N-1} (N-\rho) \frac{\sin\left(\frac{2\pi d}{\lambda} \rho\right)}{\frac{2\pi d}{\lambda} \rho}},$$

which is a number solely dependent on array geometry.

We can write the signal gain, Eq. 5, for an unweighted array as

$$G_S = \frac{\left(\sum_{i=1}^N v_i \right)^2}{\sum_{i=1}^N v_i^2} \times \frac{\text{D.I.}}{N} \quad (\text{Eq. 6})$$

We note that for the special case of hydrophone spacing equal to a multiple of half-wavelengths ($d=m\lambda/2$), the factor $\frac{D.I.}{N} = 1$, and the signal gain is simply

$$G_S = \frac{\left(\sum_{i=1}^N v_i \right)^2}{\sum_{i=1}^N |v_i|^2} \quad (\text{Eq. 7})$$

for $\lambda/2$ -spaced hydrophones.

2 THE SIGNAL FIELD

The SACLANTCEN normal-mode acoustic propagation model (SNAP) was used extensively in this study to provide the acoustic field incident on the array from a distant point source for many sets of conditions [2]. This model represents the medium as composed of three fluid layers: the water column, the upper layer of the sea bed, and the remaining sea bed extending to infinite depth. The sea surface and all interfaces are considered flat, and densities within each layer are considered constant. The speed of sound can vary with depth in the first two layers.

For this study the model was used in its range-independent form (sound-speed profile and bottom conditions not changing with range). The model calculates wave numbers and eigenfunctions of the discrete set of normal modes that are solutions of the depth-dependent part of the wave equation for specified conditions. (The continuous part of the model spectrum arising from branch line integrals is not included — these are only important at close range.)

The acoustic pressure field due to mode m at the point (r,z) is given by

$$P_m(r,z) = \frac{\rho_0}{H} \sqrt{\frac{2}{\pi r}} \frac{u_m(z_0) u_m(z)}{\sqrt{k_m}} e^{-\alpha_m r} e^{ik_m r - \pi i/4}, \quad (\text{Eq. 8})$$

where z_0 = source depth
 k_m = wave number of mode m
 H = water depth
 α_m = modal attenuation coefficient
 u_m = modal eigenfunction

and the time dependent factor $\exp(-i\omega t)$ has been omitted.

The modal attenuation coefficient is made up of two contributions representing attenuation of compressional waves in the two layers of the bottom (shear waves are not included in this study). These depend on mode number, sound speed in the layer and, as input data, attenuation coefficients for each layer. The coefficients are directly proportional to frequency and are input in units of dB/wavelength. The figures used in this study were for the bottom in the region southeast of Elba. They were obtained from [3], where the precise values of the coefficients were determined in order to give best agreement between theoretical transmission loss calculated from the SNAP model and experimentally-measured values.

The output form of the model used was pressure amplitude and phase of each mode at each hydrophone position. The total acoustic field at any point is the phase-coherent sum of the modes. This quantity gives the instantaneous pressure, but fluctuates with small changes in range, depth and frequency as the relative phases of the modes change. For example, a 180° phase change between the lowest and highest modes (for a particular set of conditions at a frequency of 500 Hz) occurs within a change of range of a few tens of metres. Thus particularly large fluctuations occur at short ranges (less than 5 km say) where high-order modes make substantial contributions, but even at ranges over 10 km the two or three strong modes remaining create a fluctuating total field. For the purposes of evaluation and comparison of signal gains for many conditions, an alternative method of combining the modal field is by incoherent addition. For a particular set of conditions this gives an average result that is not subject to instability. This is a purely theoretical procedure: the actual field existing in the water and detected by the array is of course the coherently-combined modal field. Comparison with experiment can be made by averaging the experimental results over time as range, or other parameters, change.

3 METHODS OF COMBINING HYDROPHONES

To obtain signal gain, G_S , from Eq. 6 the hydrophone voltages must be combined in some manner. One method is simply to add them in phase (i.e. with the w_i 's = 1), as in conventional beamforming, forming a beam perpendicular to the array. Another approach, called mode-matching, is to attempt to take advantage of the nature of the incident field by matching the array to the shape of the most powerful mode by means of amplitude and phase weightings applied to the hydrophones. This results in an enhancement of the response of the array to this mode and a tendency to reject the other modes. This effect is due to the orthogonal properties of normal modes.

The above two methods are described in the two sections of this chapter, and expressions for the respective signal gains are given relative to the 'average' hydrophone of the array. It is also worth calculating how much the power output of the 'maximum hydrophone' differs from this average value. There may be certain conditions in which the signal gain of the array is very low or even negative; in such cases one would do better to simply select the hydrophone with the largest power output.

3.1 Beamforming Method

For this theoretical treatment we take each mode separately and add the hydrophone outputs in phase to obtain the array voltage due to each mode. We then combine the modal contributions incoherently to give array power output. As previously mentioned, this is a way of obtaining the average power of an array looking in the forward direction.

Equation 8 gives the complex pressure due to mode m . We require the sum over the different depths z_i of the N hydrophones for fixed range r for the numerator of Eq. 6 for G_S :

$$\text{i.e. Array voltage due to mode } m = \sum_{i=1}^N P_m(r, z_i) ,$$

taking the $w_i = 1$, all hydrophones have the same response equal to unity.

The total power output of the array due to all modes is, by incoherent addition,

$$(\text{signal power})_{\text{array}} = \sum_m \left(\left| \sum_{i=1}^N P_m(r, z_i) \right|^2 \right) .$$

The denominator of Eq. 6 also obtained by incoherent mode addition is

$$\sum_{i=1}^N \sum_m |P_m(r, z_i)|^2 ,$$

and therefore, from Eq. 6, signal gain is

$$G_S = \frac{\sum_m \left(\left| \sum_{i=1}^N P_m(r, z_i) \right|^2 \right)}{\sum_{i=1}^N \sum_m |P_m|^2} \times \frac{D.I.}{N} \quad (\text{Eq. 9})$$

If the incident field was a single plane wave normally incident on the array we would have $m = 1$ and the pressure P_m would have constant phase over the array; Eq. 8 would reduce to

$$G_S = \frac{(NP_m)^2}{NP_m^2} \times \frac{D.I.}{N} .$$

For the case of half-wavelength spaced hydrophones, the factor $D.I./N = 1$, and the signal gain in dB is the well-known $10 \log N$.

3.2 Mode-matching Method

This approach utilizes the orthogonal properties of the modal eigenfunctions $u_m(z)$, which appear in the expression for the pressure field $P_m(r,z)$. These functions satisfy the depth-dependent Helmholtz equation, together with boundary conditions at the interfaces of the three layers modelled as given in [4]. It can be shown from eigenfunction theory [5] that the following orthogonality relation holds:

$$\int_0^{\infty} \rho(z) u_\ell(z) u_m(z) dz = \delta_{\ell m} ,$$

where ρ is the density, and takes its appropriate values in the three layers modelled, being a constant within each layer.

From Eq. 8 we can write, for fixed range r :

$$\int_0^{\infty} \rho P_\ell(z) P_m(z) dz = \text{constant} \times \delta_{\ell m} ,$$

(i.e. = 0 unless $\ell = m$)

the integration being over the total water column and sea bed.

Hence, if we choose weighting factors, w_i , for the hydrophones proportional to the density and to the pressure of a selected mode M , say, at the appropriate depth (e.g. see Fig. 1). i.e. $w_i \propto \rho(z_i) P_M(z_i)$, then the response of the array to this M^{th} mode is

$$\sum_{i=1}^N w_i P_M(z_i) \propto \sum_{i=1}^N \rho(z_i) P_M^2(z_i)$$

and the response to any another mode, L , is

$$\propto \sum_{i=1}^N \rho(z_i) P_M(z_i) P_L(z_i) .$$

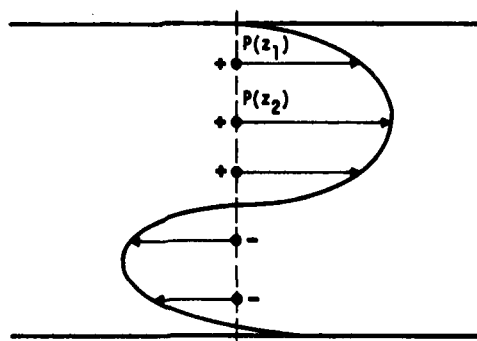


FIG. 1 EXAMPLE OF WEIGHTINGS TO MATCH TO A THIRD ORDER MODE

From the orthogonality relation we see that if our array were infinite and continuous, all modes other than the selected one would be completely rejected by it, and the response to the selected mode M maximized. For a finite array of discrete hydrophones some degree of rejection and enhancement occurs. This has been demonstrated experimentally by Ferris [6 and 7] for the purpose of detection and resolution of individual modes. Here we use this method to improve array performance by matching the array to the most powerful mode existing under the prevailing conditions. Modes close (in arrival angle) to the selected one will suffer less rejection than distant ones. A short array will accept a large group of modes centred around the selected one; this group becomes smaller as the array gets longer and is more able to discriminate between modes.

Since the density ρ is considered constant within the water column, the weighting factors w_i , when matching the array to mode M , are given by

$$w_i = \beta P_M(z_i) ,$$

where β is a constant of proportionality (the same for all hydrophones).

In order to compare with the previous method (beamformer), where all hydrophones had unit responses, we require, for normalization:

$$\sum_{i=1}^N w_i^2 = N ,$$

therefore

$$\beta^2 = \frac{N}{\sum_{i=1}^N |P_M(z_i)|^2} .$$

The response of the array matched to mode M to the arrival of mode ℓ is

$$\tau_{M_\ell} = \sum_{i=1}^N \beta P_M(z_i) P_\ell(z_i) ,$$

and combining the effects of all modes incoherently we find (for the numerator of Eq. 6)

$$\text{Array power output} = \sum_{\ell} \tau_{M_\ell}^2 = \beta^2 \sum_{\ell} \left(\sum_{i=1}^N P_M(z_i) P_\ell(z_i) \right)^2 .$$

We can now write the full expression for the signal gain by mode-matching from Eq. 5 by substituting for the w_i 's. For the case of $\lambda/2$ -spaced

hydrophones a simpler form is obtained since the terms $\sin(\frac{2\pi d}{\lambda}(1-j))$ are all zero, and we have

$$G_S = \frac{N \sum_{\ell} \left(\sum_{i=1}^N P_M(z_i) P_{\ell}(z_i) \right)^2}{\sum_{i=1}^N P_M^2(z_i) \times \sum_{i=1}^N \sum_{\ell} |P_{\ell}|^2} \quad (\text{Eq. 10})$$

The expression reduces to the ideal $10 \log N$ result if only one mode exists and we match to it i.e. only mode M exists and we have

$$G_S = \frac{N \left(\sum_{i=1}^N P_M^2(z_i) \right)^2}{\sum_{i=1}^N P_M^2 \times \sum_{i=1}^N P_M^2} \quad \text{for } \lambda/2\text{-spaced hydrophones.}$$

$$= N.$$

In order to 'mode-match' in practice, prior knowledge of the incident field would be required for evaluation of the appropriate hydrophone weightings (this would at least involve an estimate of bottom type, sound-speed profile and source frequency).

Normally for propagation from long ranges only the first few modes are of interest, since they are likely to contain most of the energy. A several-channel 'modeformer' (analogous to a beamformer), could be used to match the array to these modes. The channel with the largest output could then be selected.

In theory, a modeformer with enough channels to match all existing modes should extract all available energy from the water. To make use of this, however, requires combination of the channels, and this can be done coherently only if range is known in advance.

4 RESULTS: VARIATION OF SIGNAL GAIN WITH ARRAY LENGTH

Equations 9 and 10, for the signal gain by beamforming and mode-matching respectively, have been evaluated by a computer program that uses the SNAP output of amplitudes and phases of the modes over depth as its source of data. This program outputs the signal gains as functions of number of hydrophones and also gives the ratio of maximum hydrophone power to average hydrophone power.

Figure 2 shows results for the particular case of 500 Hz frequency, 10 km range, and a winter (slightly upward-refracting) sound-speed profile, with the source and the array centre at mid-water depth. We see that the beamformer signal gain increases with the number of $\lambda/2$ -spaced hydrophones until a maximum is reached with about 30 hydrophones.

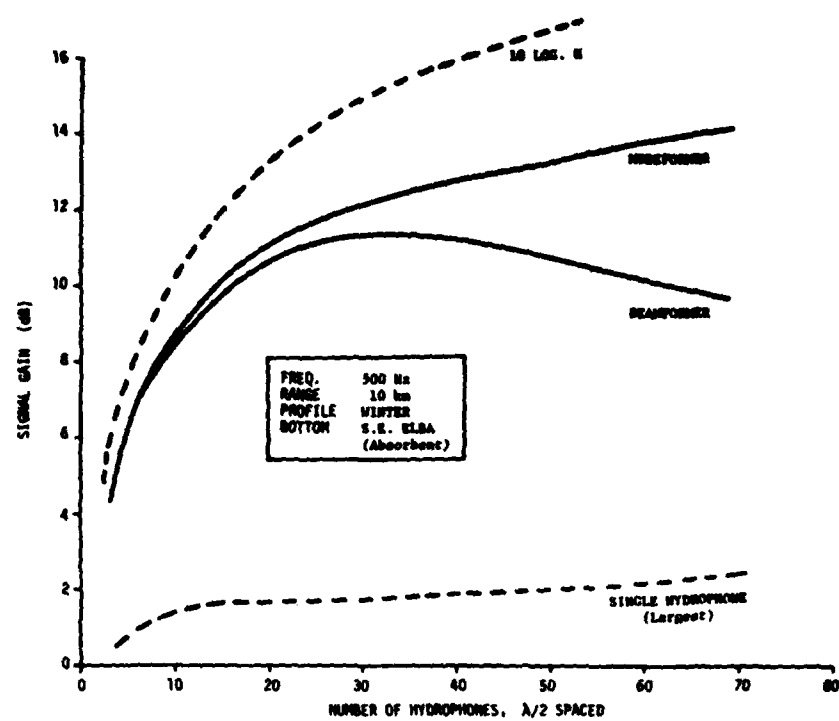


FIG. 2 SIGNAL GAIN VS. NUMBER OF HYDROPHONES

after which it falls off for longer arrays. The mode-matching curve shows gains always equal to or above those of the beamformer, and signal gain continues to increase until the whole water column is filled with hydrophones. The rate of increase of signal gain by mode-matching is less than the ideal $10 \log N$ rate, also shown in Fig. 2, being about 2dB per doubling of array length in this particular case.

The modes incident on the array can be considered as having 'angles of incidence' in the vertical plane given by $\pm \cos^{-1}(k_m / \frac{\omega}{c(z)})$, where c is the sound speed at depth z . The higher the mode number, m , the higher is the angle of incidence, but generally all modes are contained within $\pm 20^\circ$ of the horizontal (except in some extreme circumstances, e.g. perfectly rigid bottom conditions). The performance of the beamformer can be explained by considering the modes in this way. As array length is increased, the broadside beam narrows until a point is reached when an important mode is excluded from main-beam detection and signal gain then begins to decrease with the addition of more hydrophones and further narrowing of the beam. The maximum signal gain from Fig. 2 occurs at about 30 hydrophones or an array length of 15λ , which corresponds to a main beamwidth of 8° between nulls at $\pm 4^\circ$. The modal structure for these conditions of profile, source, and receiver depths, shows that the most powerful mode, number 3, is included in this beamwidth, having an angle of incidence of 1.5° . The second most powerful mode, number 5, incident at 3.5° , is substantially suppressed and the third largest mode (number 7) is excluded, being incident at 4.8° .

From Fig. 2 we see that for arrays up to about 8λ in length, there is little difference between the two processing methods and the signal gain is within 3 dB of the $10 \log N$ value. The fourth curve of Fig. 2 shows the difference between the largest single hydrophone output and the average hydrophone power. For long arrays this is about 2.5 dB and in these circumstances much below the signal gains of the array by either processing method.

It will be seen later that the beamformer performs in much the same way under most circumstances, i.e. the maximum signal gain obtainable occurs with an array length of 15λ to 30λ . In the case given here this represents an array occupying about a third of the 120 m water column.

5 EFFECT OF PARAMETERS

The main parameters that affect the signal gain and its variation with array length in shallow water are:

- Range.
- Frequency.
- Bottom type.
- Sound-speed profile.
- Source depth.
- Array position in water column.
- Water depth.

Sections 5.1 to 5.7 of this chapter study each of these in turn for an array of $\lambda/2$ -spaced elements. Arrays with different, but equal, element spacing, varying from $\lambda/4$ to 2λ are investigated in Sect. 5.8. Spacing affects the signal gain in two ways (see Eqs. 9 and 10): via the factor $D.I./N$, which arises from the isotropic noise postulate and is only equal to 1 for spacings that are multiples of $\lambda/2$; and also from the signal point of view since arrays with the same number of hydrophones but of different physical lengths sample different portions of the water column.

All results in the following refer to the case of the source and the centre of the array located at mid-water depth, unless otherwise stated.

5.1 Range

Signal gain increases with increasing range for both processing methods, as shown in Fig. 3 for ranges between 10 km and 30 km. The higher-order modes are more attenuated with increasing distance from the source and thus the acoustic field, consisting mainly of a few modes, is more coherent. Figure 3 shows signal gain improvements of up to 4 dB between ranges of 10 and 30 km. For all ranges the maximum gain by beamforming is achieved with an array of about 15λ in length. At the higher range, gain remains at more or less the same value as the array becomes longer, whereas at lower ranges gain decreases beyond the 15λ length. This is due to the wide angular incidence of energy at low ranges, some of which is rejected by the main beam of the array. At higher ranges most of the energy is contained in the first few modes and continues to be received by longer arrays.

5.2 Frequency

The number of modes of propagation and the degree of attenuation by the bottom are strongly frequency-dependent quantities. (The former is directly proportional to frequency.) Thus an increase of frequency leads to two conflicting effects as far as signal gain is concerned. An incident field of many modes gives lower signal gains; whereas increased attenuation, particularly of the higher orders of these modes, improves signal gain.

A third effect when studying $\lambda/2$ -spaced arrays is that the physical length of the array varies with frequency, for higher frequencies: a shorter array samples a smaller and more coherent portion of the water column.

Figure 4 shows results for signal gains for 100 Hz, 550 Hz and 950 Hz for an array with hydrophone spacing adjusted to $\lambda/2$ in each case. We see that at 100 Hz the water column is completely filled with hydrophones before any gain limitation is reached. Signal gain increases with frequency; a 3 dB increase between 550 Hz and 950 Hz is found for both processing methods. The number of the largest mode changes with frequency as indicated against the mode-matching curves.

The variation of signal gain with number of hydrophones for an array with one fixed hydrophone spacing is shown in Fig. 5 for three frequencies

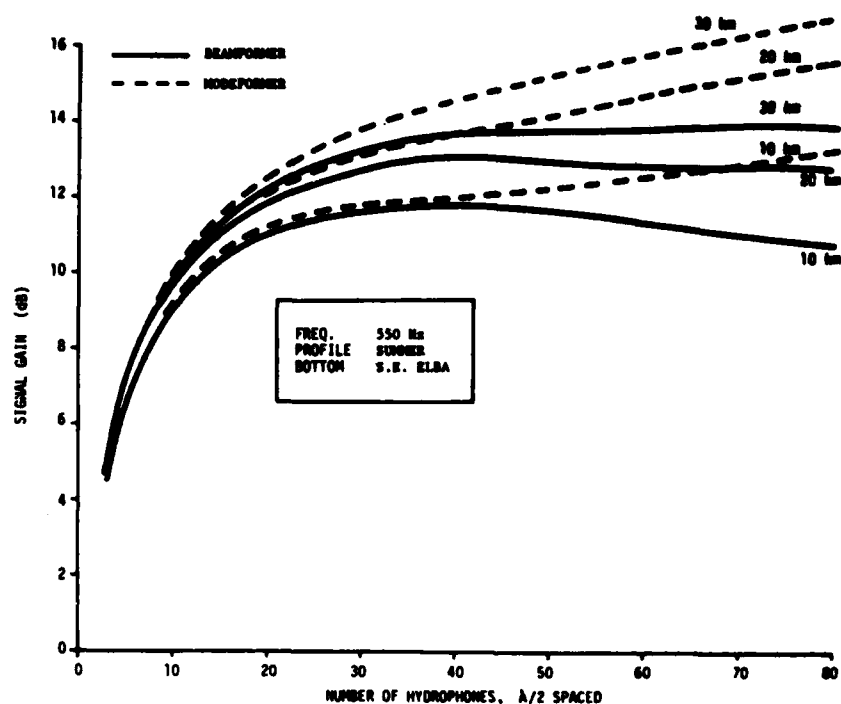


FIG. 3 EFFECT OF RANGE ON SIGNAL GAIN

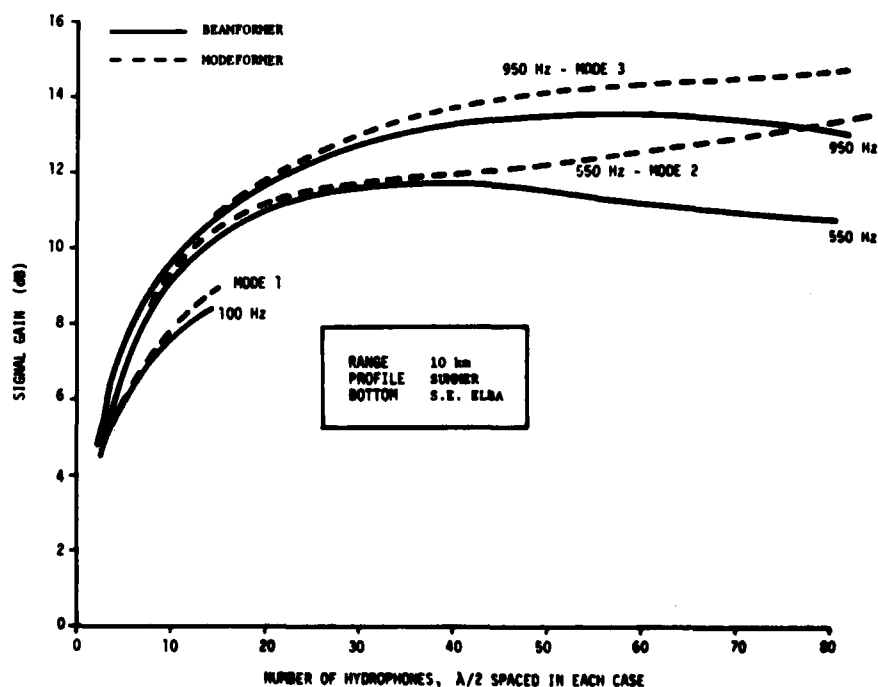
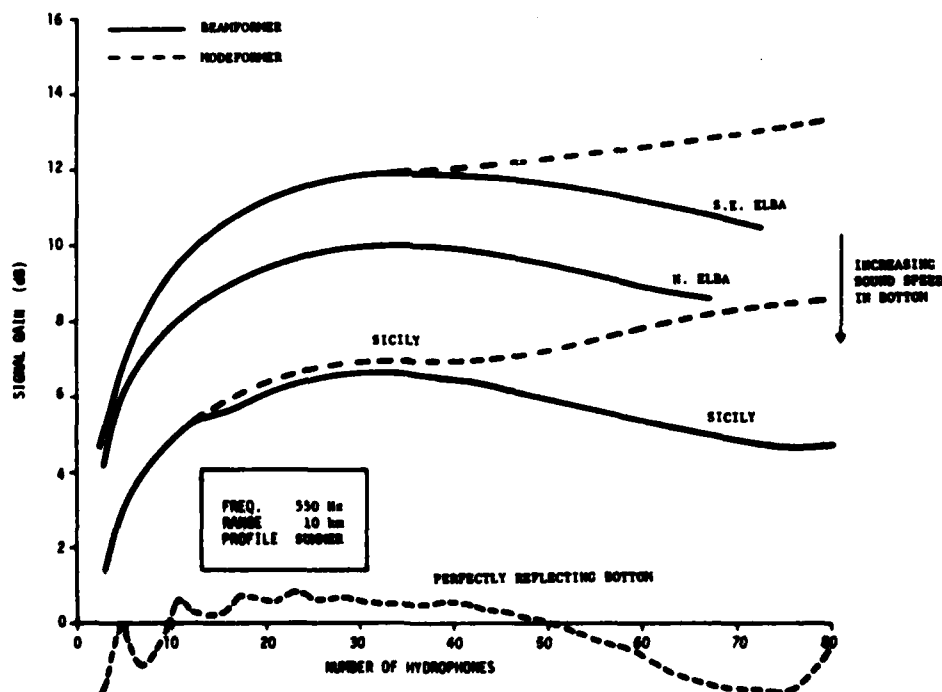
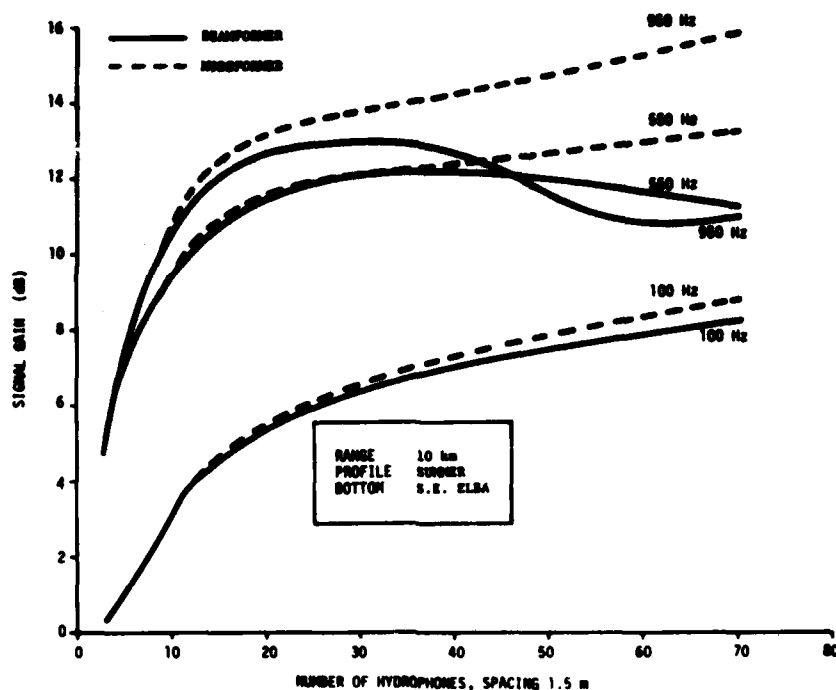


FIG. 4 EFFECT OF FREQUENCY ON THE SIGNAL GAIN OF A HALF-WAVELENGTH SPACED ARRAY



(100, 550, 950 Hz). The spacing used is 1.5 m, which is approximately $\lambda/2$ for 550 Hz. (This spacing and these three frequencies were used in our first set of experiments at sea.) The factor $D.I./N$ of Eqs. 9 and 10 for signal gain is now not equal to 1, and its appropriate values have been included in the calculations.

We see that for 950 Hz, when the spacing is near to λ , the signal gain by beamforming is higher than in the $\lambda/2$ case of Fig. 4 for arrays of less than 30 hydrophones, and lower for longer arrays.

For 100 Hz the spacing is around $\lambda/10$ and gives a much slower increase of gain with number of hydrophones than the $\lambda/2$ case. The effects of different hydrophone spacings are discussed more fully in Sect. 5.8.

The effect of frequency is also investigated in connection with different water depths in Sect. 5.7.

5.3 Bottom Type

Signal-gain calculations have been carried out for three sets of bottom conditions obtained from data collected southeast of Elba, north of Elba and off Sicily. In the S.E. Elba region the bottom consists of a low-speed 6 m layer of clay above sand, the sound speed at the water interface being 1470 m/s. In the N. Elba region the speed in the upper layer of the bottom is between 1500 and 1600 m/s and in the Sicily region about 1700 m/s. The higher the sound speed in the bottom the more energy is reflected back into the water. Thus in the Sicily region the higher-order modes suffer little attenuation compared with those in the S.E. Elba region where most of the energy is contained in the first few modes (and within small angles of incidence) once ranges of several kilometres have been reached.

The result is that signal gains are low in the highly-reflecting conditions. Since more energy is spread over wider angles of incidence, the beam of the beamformer rejects a considerable proportion of the total energy as it narrows with increasing array length (whereas the 'average hydrophone' receives all modes).

Also in the mode-matching method, matching to one mode causes partial rejection of other modes, which is of more importance in the Sicily conditions.

Figure 6 shows the gain obtained for the three types of bottom. The general shapes of the curves remain the same, the beamformer having a maximum gain at a length of about 15λ in each case. However gains for Sicily are 5 to 6 dB below the S.E. Elba results and the N. Elba results are about 2 dB below those of S.E. Elba. The mode-matching results are similarly affected. Also added to Fig. 5 is the beamformer result for the case of a perfectly rigid bottom (i.e. where the sound speed in the bottom is infinite and total reflection occurs). Very low and often negative gains are obtained, the energy from the 84 modes existing under these conditions being spread over angles of incidence of up to 87° . This is one of the few cases when selection of the largest hydrophone output gives better results than the beamformer. The maximum hydrophone power was calculated to be 1.7 dB above the average hydrophone, which is a higher gain than the beamformer gain for all array lengths.

5.4 Sound-speed Profile

Three sound-speed profiles obtained during sea tests have been used to investigate effects on signal gains. These are a winter and a summer profile from the Elba area, and a second summer profile (summer 2) taken in the Strait of Sicily. This latter is an example of an extreme thermocline gradient.

The results for signal gain by beamforming are plotted for these three cases in Fig. 7. Sound-speed profile influences both mode shapes and "arrival angles", but, at least in this case of a mid-depth source, signal gains differ only by up to 3 dB between the three cases. The maximum signal gain by beamforming occurs around the 15λ length in all cases. Signal gain by modeforming is similarly affected.

The fourth curve of Fig. 7 is for an isovelocity profile which shows higher gains for longer arrays, the maximum gain occurring with an array length of 25 to 30λ . Under these conditions arrival angles of the modes remain constant over the dimensions of the array.

5.5 Source Depth

The depth of the source determines the extent to which the modes are excited relative to each other. Thus the mode numbers of the most powerful modes change with source depth. This parameter can have a large effect on signal gains.

Figures 8 and 9 give results for summer and winter conditions respectively. In the case of the source positioned in a steep sound-speed gradient, drastic reduction in gain by beamforming can occur as shown for the 30 m source of Fig. 8. For source depths below the thermocline, little change is found. Signal gain by modeforming is less affected provided the largest mode is selected corresponding to each source depth. For the 30 m source, Fig. 8 shows that the advantage of mode-matching over beamforming can be considerable: up to 10 dB for array lengths of 10λ to 20λ . Also shown in Fig. 7 is the maximum hydrophone to average hydrophone gain for the shallow source, showing slight advantage over beamforming for these same array lengths.

In the case of the slightly positive gradient of the winter profile of Fig. 9, source depth still has a large effect. Here the deep source gives the worst performance for the beamformer.

5.6 Array Depth

The depth of the receiver affects the relative amplitudes of modes as seen by the array, and, in non-isovelocity conditions, their arrival angles. Signal gain is therefore influenced by the position of the array in the water column. Figure 10 shows results for the array at mid-water depth, near the surface, and near the bottom. In the latter two cases array length is physically limited by the surface and the bottom, and both have worse results than the mid-depth array by up to 2 dB and 4 dB respectively. These results are for a winter profile, but

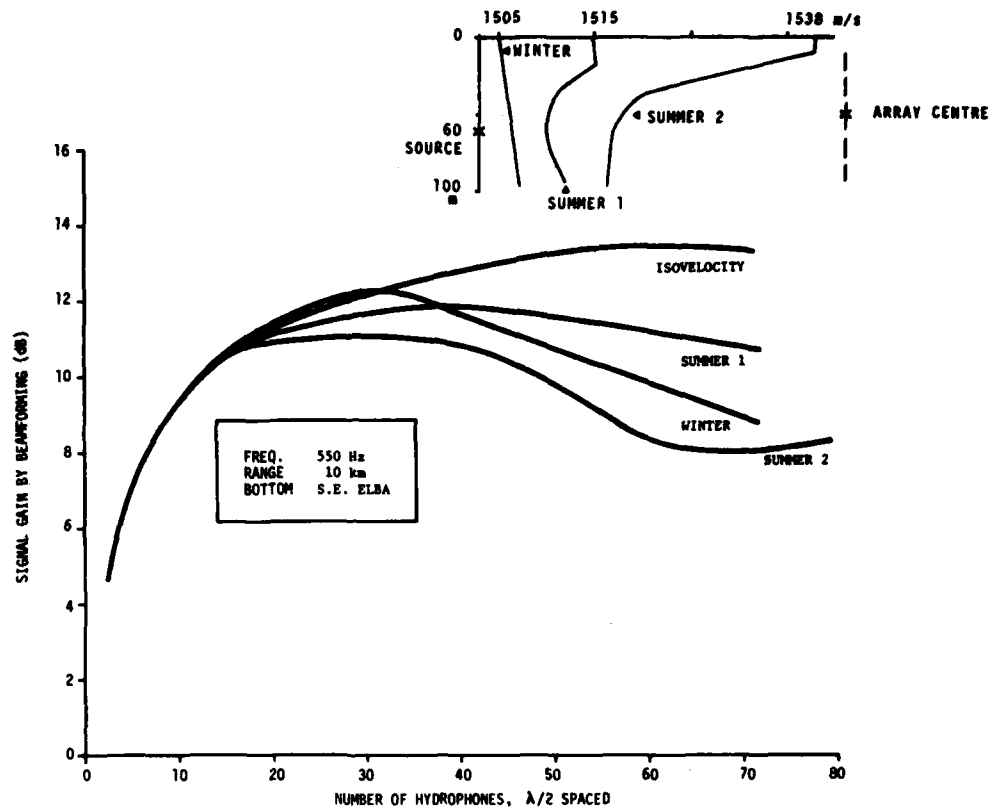


FIG. 7 EFFECT OF SOUND SPEED PROFILE

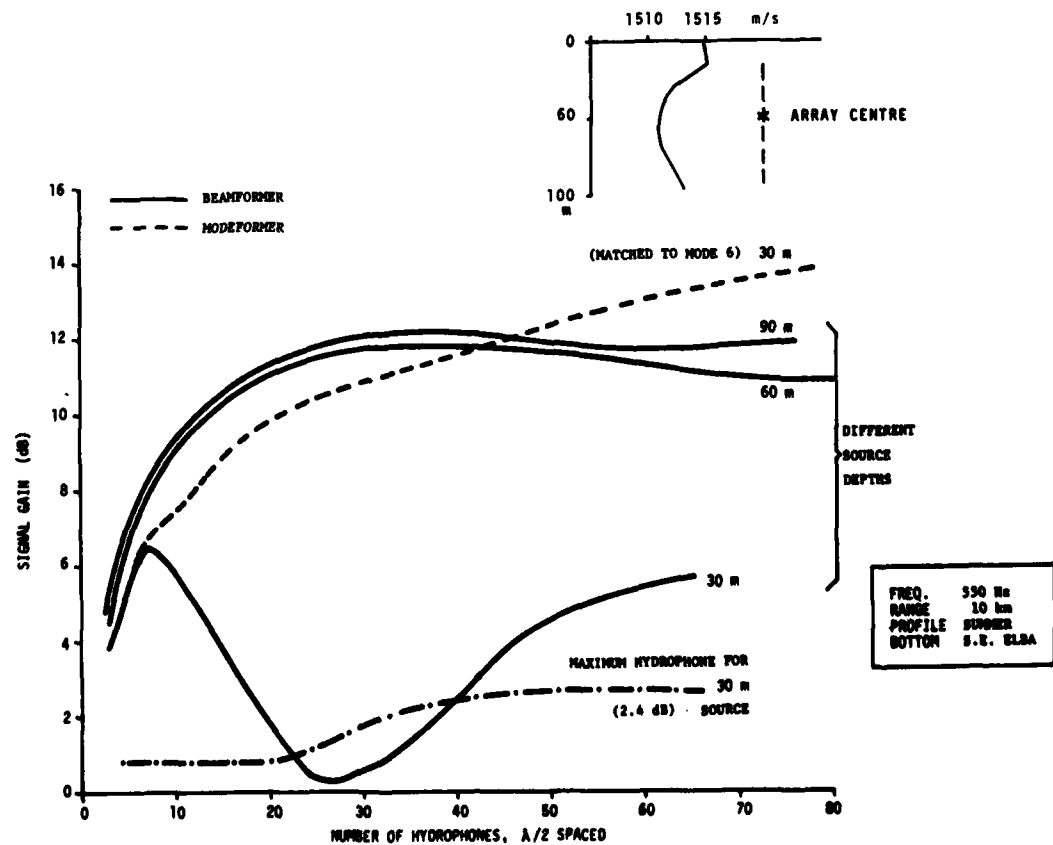


FIG. 8 EFFECT OF SOURCE DEPTH - SUMMER PROFILE

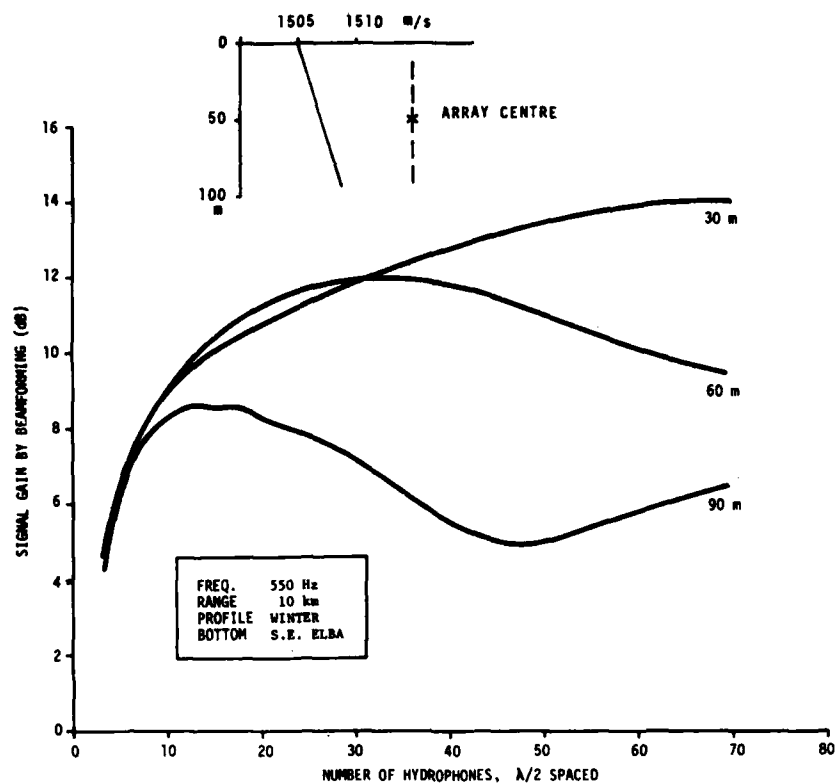


FIG. 9 EFFECT OF SOURCE DEPTH — WINTER PROFILE

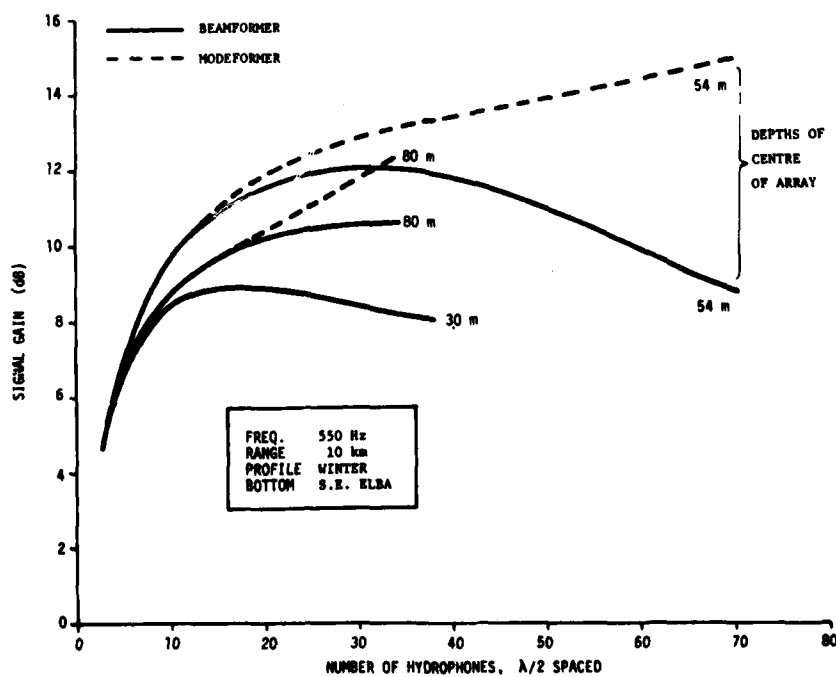


FIG. 10 EFFECT OF ARRAY DEPTH

similar ones are obtained for summer conditions. The array is best placed near the middle of the water column, well below any thermocline and away from surface and bottom effects.

5.7 Water Depth

The number of propagation modes that exists is directly proportional to both water depth and frequency. Increases in water depth, h , for fixed frequency leads to a more incoherent incident field and lower signal gains. Figure 11 gives results for three different water depths for a frequency of 500 Hz, and shows differences of 2 to 3 dB between depths of 50 m and 200 m.

If we vary both water depth and frequency such that the ratio h/λ remains constant, we find that the number of modes does not change but signal gains still differ considerably. Figure 12 shows signal gains by beamforming for three sets of frequency/depth conditions, which each give rise to 23 modes. Gain is plotted against number of hydrophones with spacings adjusted to $\lambda/2$ for each frequency. Arrays of the same number of hydrophones thus sample the same proportion of their respective water columns. We see much higher gains for the highest frequency/shallowest water condition. This is due to the greater attenuation of higher order modes at higher frequency. Examination of the mode structure shows that the largest mode for the 1000 Hz/50 m case is over 12 dB above the next largest. For the 100 Hz/500 m case there are 10 modes spread over arrival angles up to $\pm 15^\circ$ which are within a few dB of each other, giving a much more incoherent field than the first case.

5.8 Variation of Hydrophone Spacing

It is important to know how best to distribute a given number of hydrophones over the water column. (This number may be limited by cost, number of channels available for data transmission, etc.)

Half-wavelength spacing is commonly used. It results in zero correlation of isotropic noise between hydrophones and also eliminates grating lobes. This spacing reduces the factor $D.I./N$ of Eqs. 9 and 10 to 1. However, we see that if $D.I./N > 1$ array performance could be improved. This would arise with spacings that give negative correlations for isotropic noise. In this way the noise field is used to advantage to increase "signal gain".

Considering spacings between $\lambda/4$ and λ , the maximum value of $D.I./N$ occurs in the region of 0.7 to 0.9λ , depending on N ; thus from the noise point of view this would be the best spacing.

However, sampling the signal field at different points also has its effect on signal gain. Wider-spaced arrays span more of the water column and in shallow water this usually means that signal correlation and hence gain are reduced.

Figure 13 gives signal gain vs number of hydrophones for different spacings between $\lambda/4$ and 2λ for a frequency of 500 Hz. For the

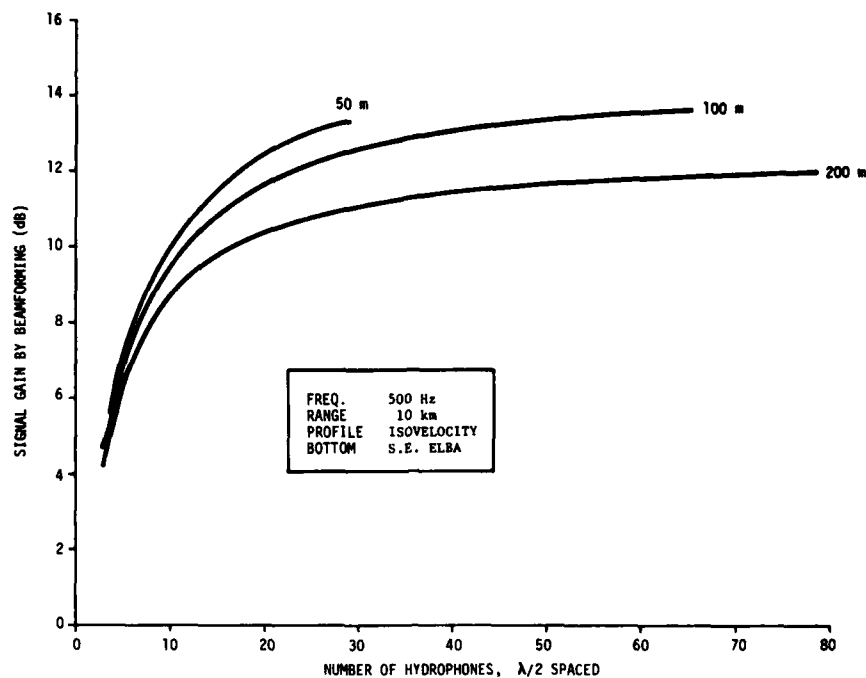


FIG. 11 EFFECT OF WATER DEPTH

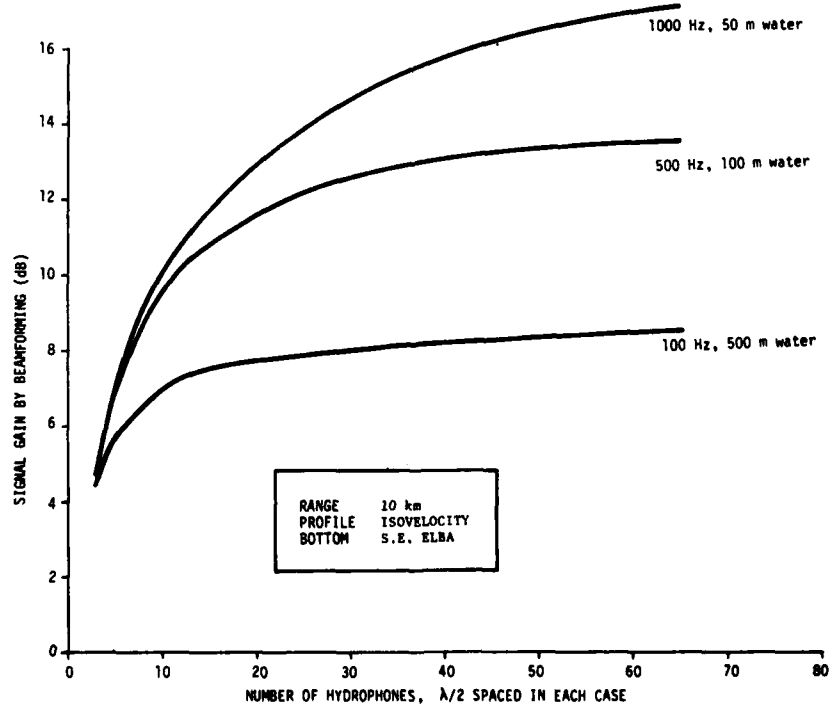


FIG. 12 SIGNAL GAIN FOR DIFFERENT FREQUENCY/WATER DEPTH COMBINATIONS

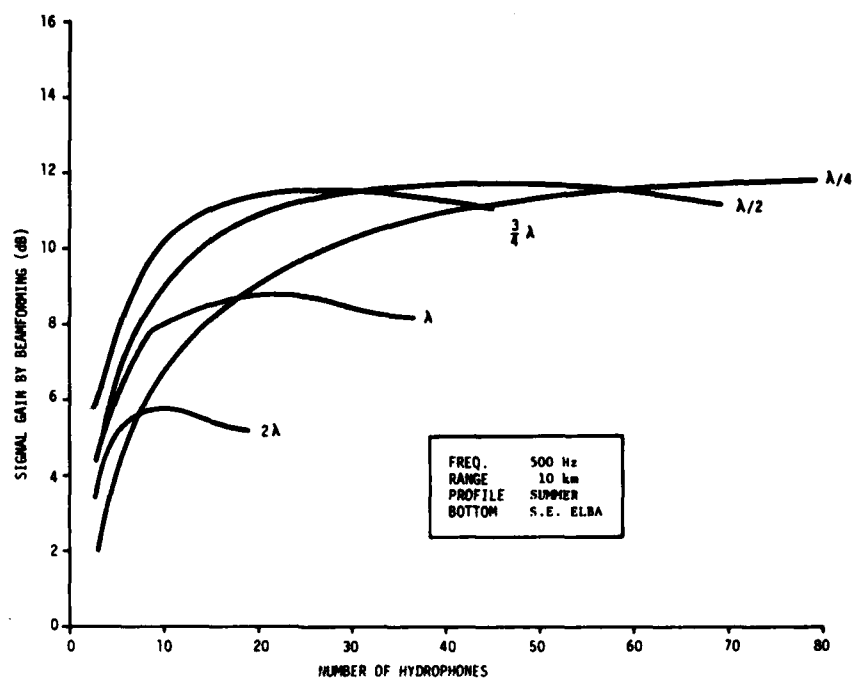


FIG. 13 SIGNAL GAIN BY BEAMFORMING FOR DIFFERENT HYDROPHONE SPACINGS

three cases that are multiples of $\lambda/2$ the D.I./N factor is equal to 1 and the differences in signal gain are due solely to the different sampling of the signal field. For the $\lambda/4$ and $3\lambda/4$ cases, both effects due to isotropic noise and signal sampling contribute to the differences in signal gain.

The gain varies considerably for different spacings — by up to 8 dB. The spacing giving best performance changes with number of hydrophones, N , and we see that:

for	$N < 30$	$3\lambda/4$ spacing gives highest gains
	$30 < N < 60$	$\lambda/2$ spacing gives highest gains
	$N > 60$	$\lambda/4$ spacing gives highest gains

For these three conditions the maximum gain of 11.7 dB is obtained. The determining factors to achieve this are an array length of 15 to 20λ , together with adequate sampling of the signal over the aperture. This latter condition is not fulfilled by the λ and 2λ spacings as shown. Note that the closest spacing of $\lambda/4$ gives no advantage in terms of maximum gain and requires more hydrophones to achieve it.

Similar results have been obtained for other frequencies. It should be remembered though that these results apply strictly to isotropic noise conditions and that a realistic shallow water-noise field could well change the effect of spacing.

CONCLUSIONS

Listed below are the main conclusions of this study with regard to the two processing methods investigated and the influences of the parameters. They apply to signal gain (i.e. taking the noise field as isotropic). This is one of the two factors that compose the total array gain. A realistic shallow-water noise field, including both the effects of shipping and sea state, is likely to have a significant effect on the values of array gain and on the way in which it varies with the parameters.

1. The maximum signal gain by beamforming, in most circumstances, is achieved with an array length of 15λ to 20λ . Longer arrays have lower gains. The value of the maximum gain is between 6 dB and 12 dB, depending primarily on bottom conditions and range. The circumstances under which this does not apply are certain combinations of source depth and sound-speed profile.

2. Signal gain by mode-matching increases continuously with array length and thus highest gains are achieved that an array which spans the whole water column. These gains are, however, generally only about 3 dB higher than the maximum achieved by beamforming. The rate of increase of gain by mode-matching with array length is between 1 dB and 2 dB per doubling of array length for arrays longer than 10λ .

3. For arrays less than about 8λ in length there is little difference between the two processing methods and signal gain is within 3 dB of the ideal $10 \log N$ value.

4. There is little to be achieved by mode-matching under normal circumstances. However, in conditions when the beamformer performs badly (e.g. extreme sound-speed gradients), mode-matching can be advantageous.
5. Higher signal gains are obtained at longer ranges. e.g. a 4 dB improvement in gain is found by increasing the range from 10 km to 30 km.
6. With $\lambda/2$ spacing, higher frequencies give higher gains. A 3 dB improvement is found by increasing the frequency from 550 Hz to 950 Hz.
7. Low-speed (absorbent) bottom conditions give higher signal gains than highly-reflecting bottom conditions. Differences of up to 6 dB have been found between the particular bottom types studied. This parameter appears to be one of the most important of those studied. It is also likely to have a large effect on the properties of shallow-water noise fields due to both shipping and, in downward-refracting conditions, to sea state.
8. The best array position within the water column is generally at mid-depth.
9. Higher signal gains are achieved in shallower water where fewer modes exist.
10. Hydrophones should be spaced at less than a wavelength apart (at the highest frequency of interest) in order to sample the signal field adequately. Maximum achievable gain can be obtained with the minimum number of hydrophones by spacing them at about 0.7 to 0.9λ .

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T.C. Cheston

General discussions.

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